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Electron transport in Coulomb- and tunnel-coupled one-dimensional systems

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Abstract. We develop a linear theory of electron transport for a system of two identical quantum wires over a wide range of the wire length L, unifying both the ballistic and diffusive transport regimes. The microscopic model, involving the interaction of electrons with each other and with bulk acoustical phonons, allows a reduction of the quantum kinetic equation to a set of coupled equations for the local chemical potentials for forward- and backward-moving electrons in the wires. As an application of the general solution of these equations, we consider different kinds of electrical contacts to the double-wire system and calculate the direct resistance, the transresistance, in the presence of tunnelling and Coulomb drag, and the tunnelling resistance. If L is smaller than the backscattering length l_P, both the tunnelling and the drag lead to a negative transresistance, while in the diffusive regime ($L \gg l_P$) the tunnelling opposes the drag and leads to a positive transresistance. If L is smaller than the phase-breaking length, the tunnelling leads to interference oscillations of the resistances that are damped exponentially with L.

1. Introduction

One-dimensional (1D) electron systems, such as occur in semiconductor quantum wires, are at the forefront of research in modern condensed-matter physics. In submicrometre-long quantum wires at low temperatures, the electron transport occurs in the ballistic regime [1] and the wire conductance reaches its fundamental value of $G_0 = e^2/\pi\hbar$. On the other hand, in sufficiently long wires the conductance is limited by scattering processes. If quantum-interference effects are neglected, as is the case when the inelastic scattering dominates, the conductance is given by σ/L, where L is the wire length and σ the conductivity described by the Drude expression $\sigma = e^2 n \tau_{tr}/m$, where n is the electron density, τ_{tr} the transport time, and m the effective mass of the electron. This regime is referred to as that of diffusive transport.

Modern technology allows one to create various systems comprising two quantum wires put close to each other so that the tunnelling of electrons between the wires and/or interlayer electron–electron interaction is important. Both of these effects give rise to coupling between the electron sub-systems in single wires and in that way modify their electronic properties. This renders the coupled double-wire systems a subject of interest. In the past few years, there have been experimental and theoretical studies of 1D–1D tunnelling [2, 3] and electron transport [4–22] along the wires of such systems. Investigations of the transport are mostly devoted to interlayer tunnelling in the purely ballistic regime and in connection with the idea of the electron-wave coupler [4, 5]. On the other hand, there are theoretical papers [18–22]

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describing the momentum transfer between the wires due to interlayer Coulomb interaction and the corresponding interlayer transresistance (Coulomb drag). Calculations of the Coulomb drag have been made for both the diffusive [18–20] and the ballistic [21] transport regimes, as well as for the regime in which the electron sub-systems are described by the Luttinger-liquid model [22].

Despite this progress, there is a distinct lack of description of the electron transport in coupled quantum wires. Even if we accept the concept that the electrons are described by a normal Fermi-liquid model, two important questions arise. The first one is that of how to describe the electrical properties when both tunnelling and interactions of the electrons (with each other and with impurities or phonons) are important. The second one is that of how to bridge the gap between the ballistic and diffusive transport regimes in such a description.

In this paper we present a linear-response theory of electron transport in coupled quantum wires that gives a reasonable answer to both questions stated above. We consider two parallel, tunnel-coupled 1D systems of degenerate electron gases adiabatically contacted to four equilibrium reservoirs, as shown and labelled in figure 1. This general scheme of a four-terminal device may describe both planar [2, 4, 5, 15] and vertically coupled [3, 9, 12, 17] double-wire devices. We take into account the interaction of electrons with themselves as well as that with acoustical phonons. We start from the quantum kinetic equation and finally transform it to a set of linear differential equations describing the distributions of the local chemical potentials for the systems of forward- and backward-moving electrons along the wires. The boundary conditions for such equations are dictated by the Landauer–Büttiker–Imry theory. This transformation is justified from microscopic calculations, which also give us expressions for the characteristic times, associated with the interactions involved, that enter the equations for chemical potentials.

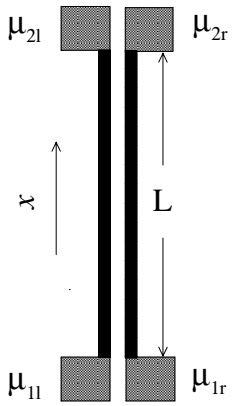

Figure 1. A schematic representation of two coupled quantum wires.

As an application of our transport theory, we analyse in detail different kinds of electrical contacts to the double-wire system. First we consider the case where the voltage is applied between the ends of one wire and calculate the 'direct' resistance of this wire as it is affected by the presence of the other one, as well as the transresistance, i.e., the resistance associated with the voltage induced at the ends of the uncontacted wire. Details about experimental measurements of the transresistance in such systems can be found in reference [23].

Next we consider the case corresponding to the tunnelling measurements [2], where the voltage is applied between the wires, and calculate the tunnelling resistance. Several previously obtained theoretical results for such quantities (some of them are for coupled 2D systems)

follow from our theory as limiting cases. A brief account of the main results appeared in reference [24].

The paper is organized as follows. In section 2 we present the microscopic model and derive the equations for the local chemical potentials in the layers. In section 3 we solve these equations, apply the results obtained to the calculation of the direct resistance, transresistance, and tunnelling resistance of the double-wire system, and describe possible transport regimes. Concluding remarks and a discussion of the approximations made are given in section 4. The appendix contains detailed microscopic calculations and expressions for the characteristic times entering the equations for chemical potentials.

2. From quantum kinetic theory to local description

Consider two homogeneous 1D quantum layers of length L, labelled left (l) and right (r), along the x-axis; see figure 1. The quantum kinetic equation for the density matrix $\hat{\rho}$ reads

$$\partial\hat{\rho}/\partial t + (\mathrm{i}/\hbar)[\hat{H}_0 + \hat{H}_C + \hat{H}_{e-ph}, \hat{\rho}] = 0. \tag{1}$$

Here we assume that electrons interact with each other via the Coulomb field $\hat{H}_C$ and with acoustical phonons via $\hat{H}_{e-ph}$. Elastic scattering is neglected, i.e., we assume ideal wires. The unperturbed Hamiltonian $\hat{H}_0$ includes both the kinetic and potential energy operators. Below, we use the basis of the isolated l- and r-layer states $F_l(y, z)$ and $F_r(y, z)$ and assume that only the lowest level is occupied in each layer. In this basis the potential energy is the matrix

$$\hat{h} = (\Delta/2)\hat{\sigma}_z + T\hat{\sigma}_x. \tag{2}$$

Here $\hat{\sigma}_i$ are the Pauli matrices, Δ is the level-splitting energy, and T the tunnelling matrix element characterizing the strength of the tunnel coupling. Such tight-binding description is often used in applications to two-level systems.

The kinetic equation can be written [25] as one for the Keldysh Green's function $\hat{G}^{-+}$. Below, we consider the case where the characteristic spatial scale of the electronic distribution is large in comparison to the Fermi wavelength $\pi\hbar/p_F$ and use the Keldysh matrix Green's function in the Wigner representation $\hat{G}^{-+}_{\varepsilon,t}(p, x)$, where p and ε are the momentum and energy and t the time. Time dependence of $\hat{G}^{-+}$ is not essential in the following, since we study a time-averaged, steady-state problem. The linear-response theory uses a Green's function of the form

$$\hat{G}^{\alpha\beta}_{\varepsilon}(p, x) = \hat{G}^{(0)\alpha\beta}_{\varepsilon}(p) + \delta\hat{G}^{\alpha\beta}_{\varepsilon}(p, x) \tag{3}$$

where α and β are + or −. The unperturbed part $\hat{G}^{(0)\alpha\beta}_{\varepsilon}(p)$ is given by

$$\hat{G}^{(0)-+}_{\varepsilon}(p) = f(\varepsilon)[\hat{G}^{A}_{\varepsilon}(p) - \hat{G}^{R}_{\varepsilon}(p)] \qquad \hat{G}^{(0)+-}_{\varepsilon}(p) = (f(\varepsilon) - 1)[\hat{G}^{A}_{\varepsilon}(p) - \hat{G}^{R}_{\varepsilon}(p)] \tag{4}$$

$$\hat{G}^{(0)--}_{\varepsilon}(p) = \hat{G}^{R}_{\varepsilon}(p) + \hat{G}^{(0)-+}_{\varepsilon}(p) \qquad \hat{G}^{(0)++}_{\varepsilon}(p) = -\hat{G}^{A}_{\varepsilon}(p) + \hat{G}^{(0)-+}_{\varepsilon}(p) \tag{5}$$

where $f(\varepsilon) = [1 + \mathrm{e}^{(\varepsilon-\mu)/k_BT_e}]$ is the equilibrium Fermi distribution function and $\hat{G}^{R,A}$ are the retarded and advanced Green's functions which satisfy the equations

$$\left[\varepsilon - \varepsilon_p - \hat{h} - \hat{\Sigma}^{R,A}_{\varepsilon}(p)\right]\hat{G}^{R,A}_{\varepsilon}(p) = 1. \tag{6}$$

Here $\hat{\Sigma}^{R,A}_{\varepsilon}(p)$ are the self-energy functions.

The linearized kinetic equation reads

$$\frac{\hbar}{2}\left\{\hat{v}_p, \frac{\partial}{\partial x}\delta\hat{G}^{-+}_{\varepsilon}(p, x)\right\} + \mathrm{i}\left[\hat{h}, \delta\hat{G}^{-+}_{\varepsilon}(p, x)\right] - \frac{\hbar}{2}\left\{\frac{\partial}{\partial x}\hat{\varphi}, \frac{\partial}{\partial p}\hat{G}^{(0)-+}_{\varepsilon}(p)\right\} + \mathrm{i}\left[\hat{\varphi}, \hat{G}^{(0)-+}_{\varepsilon}(p)\right] = \mathrm{i}\,\delta\hat{I}(\varepsilon, p, x). \tag{7}$$

Here $\{\cdots\}$ denotes anticommutators, $\hat{v}_p = \hat{P}_l v_{lp} + \hat{P}_r v_{rp}$ is the diagonal matrix of the group velocities, and $\hat{P}_l = (1+\hat{\sigma}_z)/2$ and $\hat{P}_r = (1-\hat{\sigma}_z)/2$ are the projection matrices. In this paper we consider the case of equal group velocities in the layers, with $v_{lp} = v_{rp} = v_p = p/m$. If a magnetic field is applied perpendicular to the wire plane though, v_{lp} and v_{rp} become different. Further, $\hat{\varphi}$ is the matrix of the self-consistent electrostatic potential arising due to perturbation of the electron density. In the mean-field (Hartree) approximation this matrix is diagonal. Finally, the generalized collision integral $\hat{\mathcal{I}}$ is given by [25]

$$\hat{\mathcal{I}} = -\left\{\hat{\Sigma}^{-+}\hat{G}^{++} + \hat{\Sigma}^{--}\hat{G}^{-+} + \hat{G}^{-+}\hat{\Sigma}^{++} + \hat{G}^{--}\hat{\Sigma}^{-+}\right\} \tag{8}$$

where all Green's functions $\hat{G}^{\alpha\beta}$ and self-energy functions $\hat{\Sigma}^{\alpha\beta}$ have the same arguments ε, p, and x. This corresponds to a quasiclassical description of the scattering. However, the matrix structure of $\hat{G}^{\alpha\beta}$ and $\hat{\Sigma}^{\alpha\beta}$ remains important and equation (7) is not reduced to a classical Boltzmann equation. Since we consider the interaction of electrons with each other and with acoustical phonons, the corresponding lowest-order contributions to the self-energy are given by the diagrams of figure 2. We neglect the exchange part of the Coulomb interaction for the following reasons. The first-order exchange contributions do not influence the imaginary part of the self-energy and are not, therefore, essential for the calculation of the collision integral. The second-order exchange contributions are small as compared to the second-order direct Coulomb contributions, represented by the diagram of figure 2(b), if the momentum transfer q is small in comparison with the Fermi momentum. Finally, there are no exchange contributions to the interlayer Coulomb interaction.

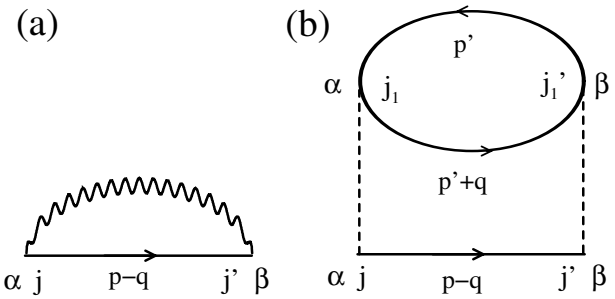

Figure 2. Feynman diagrams describing the contributions of electron–phonon (a) and electron–electron (b) interaction to the self-energies.

We consider low temperatures and degenerate electrons. We also assume that the Fermi energy is large in comparison with both the tunnelling matrix element T and the level splitting Δ, thereby neglecting the difference between the electron densities in the layers. We sum equation (7) over the electron momentum p in the regions of positive (+) (or forward) and negative (−) (or backward) group velocities and introduce the non-equilibrium part $\hat{g}_\varepsilon(x)$ of the energy distribution function in the following manner:

$$\hat{g}_\varepsilon^{\pm}(x) = \int_{\pm} \frac{\mathrm{d}p}{2\pi\mathrm{i}}\, |v_p|\, \delta\hat{G}_\varepsilon^{-+}(p,x). \tag{9}$$

Since $\delta\hat{G}^{-+}$ is essentially non-zero only in narrow intervals of energy and momentum near the equilibrium chemical potential μ and Fermi momentum p_F, we can replace $|v_p|$ in this equation by the Fermi velocity v_F, common to both layers. The integration in the + and − regions in equation (7) removes the contributions proportional to the potential matrix $\hat{\varphi}(x)$ and

we obtain

$$\pm v_F \frac{\partial}{\partial x}\hat{g}_\varepsilon^{\pm}(x) + \frac{\mathrm{i}}{\hbar}\left[\hat{h}, \hat{g}_\varepsilon^{\pm}(x)\right] = \delta\hat{I}_\pm(\varepsilon, x) \tag{10}$$

where the collision integral

$$\delta\hat{I}_\pm(\varepsilon, x) = (2\pi\hbar)^{-1}\int_\pm \mathrm{d}p\ |v_p|\,\delta\hat{\mathcal{I}}(\varepsilon, p, x)$$

depends on both $\hat{g}^+$ and $\hat{g}^-$, since it accounts for both forward- and backward-scattering processes. However, when we integrate equation (10) over the energy, the diagonal part of $\delta\hat{I}_\pm(\varepsilon, x)$ vanishes for forward-scattering contributions, and only the backscattering contributions remain; see below. In contrast, the forward-scattering contributions for the non-diagonal part of the collision integral are not eliminated by the energy integration.

The matrix kinetic equation (10) is equivalent to eight scalar equations for the four components of $\hat{g}^+$ and the four ones of $\hat{g}^-$, corresponding to forward- and backward-propagating electrons, respectively. These equations must be accompanied by boundary conditions connecting the components of $\hat{g}^\pm$ with the quasi-equilibrium distribution functions of the four leads which the quantum wires are contacted to; cf. figure 1. The distribution functions of the leads are defined by the four chemical potentials μ_{1l}, μ_{1r}, μ_{2l}, and μ_{2r}. If we assume that the potentials in the contact regions are sufficiently smooth in comparison with the Fermi wavelength but abrupt enough as compared to the characteristic scale of the electronic distribution, we can apply equation (10) in the contact region as well. It gives us the conditions of continuity for all components of $\hat{g}_\varepsilon^\pm(x)$ across the contact regions and we obtain

$$\hat{g}_\varepsilon^+(0) = -\frac{\partial f(\varepsilon)}{\partial\varepsilon}[\hat{P}_l\,\delta\mu_{1l} + \hat{P}_r\,\delta\mu_{1r}] \qquad \hat{g}_\varepsilon^-(L) = -\frac{\partial f(\varepsilon)}{\partial\varepsilon}[\hat{P}_l\,\delta\mu_{2l} + \hat{P}_r\,\delta\mu_{2r}] \tag{11}$$

with $\delta\mu_{1l} = \mu_{1l} - \mu$, etc. The forward- and backward-propagating states are 'connected', respectively, to the leads 1 and 2. The non-diagonal components vanish at the contacts because the tunnelling is absent outside the region $x = [0, L]$.

The problem described by the matrix equation (10) and the boundary conditions (11) can be considerably simplified and solved analytically if we assume that both backscattering and the interlayer tunnelling occur much less frequently than the scattering of electrons inside the layers and inside the + or − regions. The tunnelling can be made weak if, for example, the potential barrier between the wires is thick enough. As regards the backscattering, this condition is often fulfilled at low temperatures for both the electron–electron and electron–phonon scattering mechanisms. In the first case, the backscattering probability contains a factor $[K_0(2p_Fa/\hbar)]^2$, where K_0 is the modified Bessel function and a is the wire width. This factor is exponentially small for $2p_Fa/\hbar > 1$. The acoustic phonon-assisted backscattering gives a small contribution in comparison with the electron–electron forward scattering due to the smallness of the electron–phonon coupling constant. In addition, this backscattering is suppressed at very low temperatures $T_e < 2p_Fs$, where s is the sound velocity. If the stated conditions are fulfilled, the diagonal parts of the energy distribution function of the electrons have a Fermi-like energy dependence because any quasi-equilibrium Fermi function locally satisfies a kinetic equation containing the electron–electron and electron–phonon collision integrals. This means that the diagonal part of $\hat{g}_\varepsilon^\pm(x)$ is given by the following equation:

$$[\hat{g}_\varepsilon^\pm(x)]_{jj} = -\frac{\partial f(\varepsilon)}{\partial\varepsilon}\,\delta\mu_j^\pm(x) \tag{12}$$

where $\delta\mu_j^\pm(x) = \mu_j^\pm(x) - \mu$ $(j = l, r)$ do not depend on the energy. The quantities $\mu_j^\pm(x)$ have the direct meaning of *local chemical potentials* for the layers l and r. It is convenient to introduce also the non-diagonal components of the chemical potentials

$\mu_{lr}^{\pm}(x) = \mu_u^{\pm}(x) - \mathrm{i}\mu_v^{\pm}(x)$ and $\mu_{rl}^{\pm}(x) = \mu_u^{\pm}(x) + \mathrm{i}\mu_v^{\pm}(x)$, by writing the whole chemical potential matrix as

$$\delta\hat{\mu}^{\pm}(x) = \int \mathrm{d}\varepsilon\ \hat{g}_{\varepsilon}^{\pm}(x). \tag{13}$$

Below we drop the symbol 'δ' in $\delta\hat{\mu}^{\pm}(x)$ and the contact potentials $\delta\mu_{1l,r}$ and $\delta\mu_{2l,r}$ since all chemical potentials are counted from the same equilibrium value μ.

Substituting equation (12) into equation (10) and integrating the latter over the energy, we finally obtain eight coupled, first-order differential equations for the eight components of $\mu_k^{\pm}(x)$:

$$\pm\mathrm{d}\mu_l^{\pm}/\mathrm{d}x + (\mu_l^{\pm} - \mu_l^{\mp})(1/l_P + 1/l_D) - (\mu_r^{\pm} - \mu_r^{\mp})/l_D - 2t_F\mu_v^{\pm} = 0 \tag{14}$$

$$\pm\mathrm{d}\mu_r^{\pm}/\mathrm{d}x + (\mu_r^{\pm} - \mu_r^{\mp})(1/l_P + 1/l_D) - (\mu_l^{\pm} - \mu_l^{\mp})/l_D + 2t_F\mu_v^{\pm} = 0 \tag{15}$$

$$\pm\mathrm{d}\mu_u^{\pm}/\mathrm{d}x + \delta_F\mu_v^{\pm} + \mu_u^{\pm}/l_C = 0 \tag{16}$$

$$\pm\mathrm{d}\mu_v^{\pm}/\mathrm{d}x - \delta_F\mu_u^{\pm} + \mu_v^{\pm}/l_C + t_F(\mu_l^{\pm} - \mu_r^{\pm}) = 0. \tag{17}$$

Here $t_F = T/\hbar v_F$ and $\delta_F = \Delta/\hbar v_F$. The boundary conditions for all potentials follow from equations (11)–(13) and are $\mu_l^{+}(0) = \mu_{1l}$, $\mu_l^{-}(L) = \mu_{2l}$, $\mu_r^{+}(0) = \mu_{1r}$, $\mu_r^{-}(L) = \mu_{2r}$, and $\mu_{u,v}^{+}(0) = \mu_{u,v}^{-}(L) = 0$. The characteristic lengths l_P, l_D, and l_C result from the collision integral $\delta\hat{I}_{\pm}(\varepsilon, x)$, evaluated to the lowest order with respect to the tunnelling matrix element T; see the appendix for details. They are expressed, respectively, through the phonon-assisted 1D transport time [26] τ_P, the 1D Coulomb-drag time [19, 20] τ_D, and the phase-breaking time τ_C describing the suppression of tunnel coherence, as $l_P = 2v_F\tau_P$, $l_D = 2v_F\tau_D$, and $l_C = v_F\tau_C$. The transport time τ_P is common to both layers since we assume that the confining potentials for the wires l and r are almost identical. The analytical expressions for the τ_P, τ_D, and τ_C are given in the appendix. All characteristic lengths are sensitive to the temperature T and the level splitting Δ. It is essential that l_C, which is controlled by electron–electron interaction, is always much smaller than l_D and l_P. On the other hand, depending on the temperature and level splitting one can have different relations between l_D and l_P: both cases, $l_P \ll l_D$ and $l_P \gg l_D$, are possible.

Equations (14)–(17) with the stated boundary conditions give us a complete description of the electrical properties of double quantum-wire systems in a wide range of regimes starting from the purely ballistic transport regime $L \ll l_C$ to the diffusive transport regime $L \gg l_P, l_D$. The local currents flowing in the layers $j = l, r$ are expressed by

$$J_j(x) = G_0[\mu_j^{+}(x) - \mu_j^{-}(x)]/e \tag{18}$$

and the local tunnel currents are proportional to $T\mu_v^{\pm}(x)$.

Below, we present the general solution of equations (14)–(17) and describe two important cases, that of *long* systems, with $L \gg l_C$, and that of *short* systems with $L \sim l_C$. To characterize the effects of drag and tunnelling, we then consider different kinds of electrical contact to the double-wire system. First we consider a typical set-up for the drag measurements, where the current $J_r = J_r(0) = J_r(L)$ is injected in wire r ('drive wire') while no current is allowed to flow into wire l, $J_l(0) = J_l(L) = 0$, and calculate the transresistance R_{TR} defined as $R_{TR} = [\mu_{1l} - \mu_{2l}]/eJ_r$ as well as the 'direct' resistance $R = [\mu_{1r} - \mu_{2r}]/eJ_r$. Next, we turn to the tunnelling measurements [2], where the voltage is applied between the wires. We consider both the symmetric set-up, when all four ends of the wires are connected to external sources, with $\mu_{1l} = \mu_{2l}$ and $\mu_{1r} = \mu_{2r}$, and the non-symmetric one, when the voltage is applied between the ends $1l$ and $2r$ while the remaining ends are not contacted, $J_r(0) = 0$, $J_l(L) = 0$. For each of these cases we calculate the tunnelling resistances R_{Ts} (symmetric) and R_{Tn} (non-symmetric). Both of them can be defined as $[\mu_{1l} - \mu_{2r}]/eJ_T$, where J_T is the

total current injected. J_T is equal to $2J_l(0)$ and $J_l(0)$ for the symmetric and non-symmetric contacts, respectively.

3. Results

Since equations (14)–(17) are linear, their general solution is easily obtained as

$$2\mu^+_{l,r}(x) = (1+L/l_P)^{-1}\,[(\mu_{1l}+\mu_{1r})(1+(L-x)/l_P)+(\mu_{2l}+\mu_{2r})x/l_P] \pm \sum_i (A^+_i \mathrm{e}^{\lambda_i x} + B^+_i \mathrm{e}^{-\lambda_i x}) \tag{19}$$

$$2\mu^-_{l,r}(x) = (1+L/l_P)^{-1}\,[(\mu_{1l}+\mu_{1r})(L-x)/l_P+(\mu_{2l}+\mu_{2r})(1+x/l_P)] \pm \sum_i (A^-_i \mathrm{e}^{\lambda_i x} + B^-_i \mathrm{e}^{-\lambda_i x}) \tag{20}$$

$$\mu^\pm_v(x) = \sum_i (C^\pm_i \mathrm{e}^{\lambda_i x} + D^\pm_i \mathrm{e}^{-\lambda_i x}) \tag{21}$$

$$\mu^\pm_u(x) = -\sum_i \left(\frac{\delta_F}{\pm\lambda_i + l_C^{-1}} C^\pm_i \mathrm{e}^{\lambda_i x} + \frac{\delta_F}{\mp\lambda_i + l_C^{-1}} D^\pm_i \mathrm{e}^{-\lambda_i x} \right). \tag{22}$$

Here $\lambda_i = \sqrt{y_i}$ and $y = y_i$ are the solutions of the cubic equation

$$y^3 - 2y^2[l_C^{-2} - \delta_F^2 - 4t_F^2] + y[l_C^{-4} + (\delta_F^2+4t_F^2)^2 - 8(t_F/l_C)^2 + 2(\delta_F/l_C)^2 + 8t_F^2/l_C l_1] - 4t_F^2[4(t_F/l_C)^2 + (2/l_C l_1)(l_C^{-2}+\delta_F^2)] = 0 \tag{23}$$

where $l_1^{-1} = l_P^{-1} + 2l_D^{-1}$. The coefficients $A^\pm_i$, $B^\pm_i$, $C^\pm_i$, and $D^\pm_i$ are to be found from equations (14)–(17) and the relevant boundary conditions. Below we use the property $l_C \ll l_P, l_D$ and the condition of weak tunnel coupling $t_F \ll l_C^{-1}$ to simplify this procedure. Then the three roots of equation (23) are easily obtained as

$$\lambda_1 = \lambda \simeq 2(1/l_T^2 + 1/l_T l_1)^{1/2} \qquad \lambda_{2,3} = \lambda_\pm \simeq 1/l_C \pm \mathrm{i}\delta_F \tag{24}$$

where we introduced the tunnelling length $l_T = v_F \tau_T$. The tunnelling time τ_T, which contains a resonance dependence on the level splitting, is defined by

$$\tau_T^{-1} = \tau_C^{-1} \frac{2T^2}{\Delta^2 + (\hbar/\tau_C)^2}. \tag{25}$$

The root λ describes long-scale variations of the chemical potentials while $\lambda_\pm$ corresponds to short-scale variations. Accordingly, we consider the regimes that follow.

3.1. Long wires, $L \gg l_C$

This length range comprises the region from the 'pseudo-ballistic' ($l_C \ll L \ll l_P, l_D$) to the diffusive ($L \gg l_P$) regimes. All solutions containing $\lambda_\pm$ exist only in short regions in the vicinity of the contacts. They are evanescent inside the wire region and not essential in the calculation of the currents. Considering only the solutions involving λ, we find

$$2\mu^+_{l,r}(x) = (1+L/l_P)^{-1}\,[(\mu_{1l}+\mu_{1r})(1+(L-x)/l_P)+(\mu_{2l}+\mu_{2r})x/l_P] \pm (\mu_{1l}-\mu_{1r})P(L-x)/P(L) \pm (\mu_{2l}-\mu_{2r})\sinh\lambda x/P(L) \tag{26}$$

$$2\mu^-_{l,r}(x) = (1+L/l_P)^{-1}\,[(\mu_{1l}+\mu_{1r})(L-x)/l_P+(\mu_{2l}+\mu_{2r})(1+x/l_P)] \pm (\mu_{1l}-\mu_{1r})\sinh\lambda(L-x)/P(L) \pm (\mu_{2l}-\mu_{2r})P(x)/P(L) \tag{27}$$

where $P(x) = (1+2l_1/l_T)\sinh\lambda x + \lambda l_1 \cosh\lambda x$. The same expressions can be obtained from the four coupled balance equations compactly presented as

$$\pm \mathrm{d}\mu^\pm_j/\mathrm{d}x + (\mu^\pm_j - \mu^\mp_j)(1/l_P + 1/l_D) - (\mu^\pm_{j'} - \mu^\mp_{j'})/l_D + (\mu^\pm_j - \mu^\pm_{j'})/l_T = 0 \tag{28}$$

where $j = l, r$ and $j' \neq j$. These equations follow from equations (14)–(17) in the limit $L \gg l_C$, when one can neglect the derivatives $\mathrm{d}\mu^{\pm}_{u,v}/\mathrm{d}x$ in comparison to $\mu^{\pm}_{u,v}/l_C$. With equations (26) and (27) we obtain

$$R = \frac{\pi\hbar}{2e^2}\left[2 + L/l_P + (1 + l_T/l_1)^{1/2}\tanh(\lambda L/2)\right] \tag{29}$$

$$R_{TR} = \frac{\pi\hbar}{2e^2}\left[L/l_P - (1 + l_T/l_1)^{1/2}\tanh(\lambda L/2)\right]. \tag{30}$$

For $L \ll l_P, l_T$ we have $R \simeq \pi\hbar/e^2 = G_0^{-1}$, while the transresistance is given by $R_{TR} \simeq -(\pi\hbar/e^2)L[1/l_D + 1/2l_T]$. As seen, R_{TR} is small, always negative, and proportional to the wire length L multiplied by a sum of drag and tunnelling rates. If one neglects tunnelling, the resulting expression for R_{TR}, with τ_D given by equation (A.11), describes the *Coulomb drag in the ballistic regime* previously investigated [21] by Gurevich *et al*. When L increases and the electron transport becomes diffusive ($L \gg l_P$), we obtain, for $\lambda L/2 \ll 1$, $R \simeq (\pi\hbar/e^2)L[1/l_P + 1/l_D]$. This resistance, if one omits the drag contribution, is expressed in terms of the usual Drude conductivity $\sigma = L/R = e^2 l_P/\pi\hbar = e^2 n\tau_P/m$. The corresponding transresistance is

$$R_{TR} = -\frac{\pi\hbar}{e^2}(L/l_D)\left[1 - (L/L_0)^2\right] \qquad L_0 = (6l_P^2 l_T/l_D)^{1/2}. \tag{31}$$

Expressing l_D and l_T through the drag transresistivity $\pi\hbar/e^2 l_D$ and the tunnelling conductance $G_T = e^2\rho_{1D}/\tau_T = 2e^2/\pi\hbar l_T$, where ρ_{1D} is the 1D density of states at the Fermi level, one can see that equation (31) formally coincides with that obtained in reference [27], where a competition of drag and tunnelling effects was investigated for double quantum-well systems. For $\lambda L \sim 1$, the transresistance is large and comparable to the direct resistance, because a considerable fraction of the current penetrates the l layer due to tunnelling. This regime for double quantum wells has been investigated both experimentally [28] and theoretically [29]. If one neglects the drag and assumes the diffusive regime ($L \gg l_P$) with weak tunnelling ($l_P \ll l_T$), equations (29) and (30) describe the results obtained in reference [29]. For $\lambda L \gg 1$ we have $R_{TR} = R = (\pi\hbar/e^2 l_P)(L/2) = L/2\sigma$. This is the case when the current, though injected only in one layer, is equally distributed among the layers due to tunnelling.

Figure 3 shows the length dependence of the transresistance calculated for different relative contributions of the Coulomb drag and tunnelling. The transresistance is negative for small L but always changes its sign and becomes positive as L increases and the backscattering occurs more often (see also equation (31)). This behaviour can be explained with the help of the balance equation (28), which shows that the tunnelling tends to decrease the difference between $\mu_l^{\pm}$ and $\mu_r^{\pm}$ while the backscattering tends to decrease the difference between $\mu_{l,r}^{+}$ and $\mu_{l,r}^{-}$. Thus, for $\mu_l^{+}(0) = \mu_l^{-}(0)$ and $\mu_l^{+}(L) = \mu_l^{-}(L)$ the change of μ_l^{+} (μ_l^{-}), with x, is opposite to that of μ_r^{+} (μ_r^{-}) at small L and becomes the same as that of μ_r^{+} (μ_r^{-}) as L increases, leading to the change of $R_{TR} = [\mu_l^{\pm}(0) - \mu_l^{\pm}(L)]/eJ_r$ from negative to positive. This transition occurs at smaller L/l_P if the tunnelling is stronger (larger l_P/l_T) and the drag weaker (smaller l_P/l_D).

Although l_T is normally longer than l_P, the opposite condition can also be realized. A particularly interesting transport regime, corresponding to long quantum wires without backscattering, occurs in tunnel-coupled magnetic edge states [30, 31], since an edge state represents a 1D system where the electrons can move only in one direction. Assuming $1/l_P = 1/l_D = 0$ in equations (29) and (30), we obtain the result of reference [31] in the form

$$R = \frac{\pi\hbar}{e^2}\left[1 + (1/2)\tanh(L/l_T)\right] \qquad R_{TR} = -\frac{\pi\hbar}{2e^2}\tanh(L/l_T). \tag{32}$$

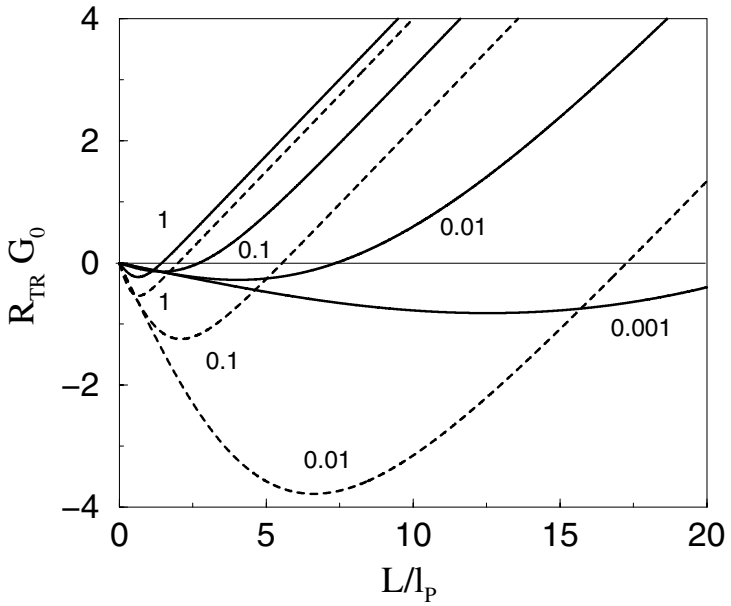

Figure 3. Transresistance R_{TR} as a function of the wire length L. The solid curves correspond to $l_P/l_D = 0.1$ (weaker drag) and the dashed ones to $l_P/l_D = 1$ (stronger drag). Each curve is marked by the value of l_P/l_T.

Consider now the behaviour of the tunnelling resistances. With equations (26) and (27) we obtain

$$R_{Ts} = \frac{\pi\hbar}{2e^2}\left[1 + (1 + l_T/l_1)^{1/2}\coth(\lambda L/2)\right] \tag{33}$$

and

$$R_{Tn} = \frac{\pi\hbar}{2e^2}\left[2 + L/l_P + (1 + l_T/l_1)^{1/2}\coth(\lambda L/2)\right] \tag{34}$$

for symmetric and non-symmetric contacts, respectively. For conditions $\lambda L/2 \ll 1$ we have $R_{Ts} \simeq R_{Tn} \simeq (\pi\hbar/2e^2)(l_T/L)$, i.e., the tunnelling resistances depend only on the ratio of the tunnelling length to the wire length. This is because the regime of $\lambda L/2 \ll 1$ corresponds to weak tunnelling and the chemical potentials $\mu_l^{\pm}(x)$ and $\mu_r^{\pm}(x)$ are close to μ_{1l} and μ_{2r}, respectively. With the use of the tunnelling conductance G_T (see above) one can rewrite the expression for the tunnelling resistances in a more transparent way: $R_{Ts} \simeq R_{Tn} \simeq (G_T L)^{-1}$. For $\lambda L/2 \sim 1$, when the coordinate dependence of the chemical potentials in the layers is important, R_{Tn} is different from R_{Ts} and both of them depend on the scattering length l_P. The drag effect is not so important as for the transresistance: the tunnelling resistances depend on l_D only if l_D is comparable to or smaller than both l_P and l_T.

Figure 4 shows the length dependence of the tunnelling resistances R_{Ts} and R_{Tn}, as given by equations (33) and (34), for several different values of the ratio l_P/l_T describing the strength of the tunnelling with respect to the backscattering. The drag effect is neglected, $1/l_D = 0$. As the wire length becomes larger than the backscattering length, the $1/L$ decrease of the tunnelling resistance changes to either an L-independent behaviour (for R_{Ts}) or to a linear increase (for R_{Tn}). In the first case the dependence on L disappears because all tunnelling occurs near the ends. In contrast, for non-symmetric contacts the resistance R_{Tn} is determined

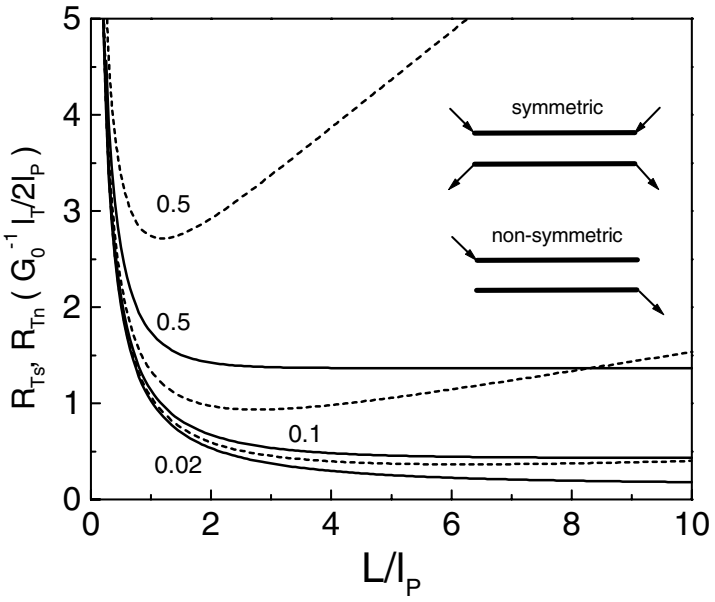

Figure 4. Tunnelling resistances for symmetric (solid) and non-symmetric (dashed) set-ups as a function of the wire length L, at $1/l_D = 0$. Each curve is marked by the value of l_P/l_T. The inset shows the currents (arrows) injected in and coming out of the wires (broad lines) for both cases.

by the Ohmic resistance of the wires instead of the tunnelling effects, and increases linearly with L. A similar effect, in applications to coupled quantum wells, is discussed in reference [29].

3.2. Short wires, $L \ll l_P, l_D$

This length range comprises the region from the purely ballistic ($L \ll l_C$) to the 'pseudo-ballistic' ($l_C \ll L \ll l_P, l_D$) regimes. Since the electrons pass along the wires almost without backscattering, R is close to $\pi\hbar/e^2$, and R_{TR} is small. However, for $L \sim l_C$ an electron tunnelling between the layers does not lose its phase memory completely and tunnel coherence effects can manifest themselves on such short lengths giving additional contributions to the transresistance R_{TR} and the tunnelling resistances R_{Ts} and R_{Tn}; accordingly the expressions for these quantities obtained in the previous subsection for $L \ll l_P, l_D$ should be modified.

A convenient analytical approach to the problem in this regime is to solve equations (14)–(17) by iterations taking $\mu^+_{l,r}(x) = \mu_{1l,r}$, $\mu^-_{l,r}(x) = \mu_{2l,r}$, and $\mu^{\pm}_{u,v}(x) = 0$ as an initial approximation. Another way is to use equations (19)–(22) directly. We obtain

$$2\mu^+_{l,r}(x) = (\mu_{1l} + \mu_{1r})(1 - x/l_P) + (\mu_{2l} + \mu_{2r})x/l_P \pm (\mu_{1l} - \mu_{1r})\,[1 - x/l_1 - 2x/l_T + 2(l_C/l_T)\Phi(x)] \pm (\mu_{2l} - \mu_{2r})x/l_1 \quad (35)$$

$$2\mu^-_{l,r}(L - x) = (\mu_{2l} + \mu_{2r})(1 - x/l_P) + (\mu_{1l} + \mu_{1r})x/l_P \pm (\mu_{2l} - \mu_{2r})\,[1 - x/l_1 - 2x/l_T + 2(l_C/l_T)\Phi(x)] \pm (\mu_{1l} - \mu_{1r})x/l_1 \quad (36)$$

where

$$\Phi(x) = [1/(l_C^{-2} + \delta_F^2)]\left[2(\delta_F/l_C)\mathrm{e}^{-x/l_C}\sin(\delta_F x) + (l_C^{-2} - \delta_F^2)(1 - \mathrm{e}^{-x/l_C}\cos(\delta_F x))\right] \quad (37)$$

is an oscillating function of the coordinate x and $\delta_F = \Delta/\hbar v_F$. Now R and R_{TR} are given,

respectively, by

$$R = \frac{\hbar\pi}{e^2}\left[1 + L/l_P + L/l_D + L/2l_T - (l_C/2l_T)\Phi(L)\right] \tag{38}$$

and

$$R_{TR} = \frac{\hbar\pi}{e^2}\left[-L/l_D - L/2l_T + (l_C/2l_T)\Phi(L)\right]. \tag{39}$$

The contribution to R_{TR} coming from the term proportional to $\Phi(L)$ is not small for $L \sim l_C$. It describes oscillations damped due to the factor $\exp(-L/l_C)$. The periodic behaviour can be described as a result of the interference of electron waves of the left and right layers along the length L: due to a finite level splitting Δ, these waves have different phase velocities.

Similar interference effects occur in the tunnelling resistances:

$$R_{Ts} \simeq R_{Ta} \simeq \frac{\hbar\pi}{2e^2}(l_T/L)\left[1 - (l_C/L)\Phi(L)\right]^{-1}. \tag{40}$$

From equation (37) for $\Phi(L)$ we see that both the transresistance and the tunnelling resistances, being functions of $\Delta L/\hbar v_F$, oscillate with level separation Δ. The oscillations are damped when the wire length L exceeds l_C, so the tunnel coherence over the wire length is suppressed.

Changing Δ by applying a voltage across the wires would lead to oscillations of R, R_{TR}, R_{Ts}, and R_{Tn}. Another way to change Δ is to apply a magnetic field B perpendicular to the plane of the wires [15, 16]. For sufficiently weak B the results presented so far still hold with the phase $\delta_F L$ having an additional contribution $2\pi\phi/\phi_0$, where $\phi_0 = h/e$ is the magnetic flux quantum and $\phi = BwL$ the flux enclosed by the area between the wires. Though the double-wire system does not form a closed current loop, this should lead to Aharonov–Bohm-type oscillations in the resistances defined by equations (38)–(40).

In very short wires, with $L \ll l_C, \delta_F^{-1}$, equations (39) and (40) become

$$R_{TR} = -\frac{\pi\hbar}{e^2}(L/l_D + L^2 t_F^2/2) \qquad R_{Ts} = R_{Tn} = \frac{\pi\hbar}{2e^2}L^{-2}t_F^{-2}. \tag{41}$$

In this regime only a small fraction of the electronic wave packet is coherently transmitted from one wire to another. The tunnelling contribution to R_{TR} follows an L^2-dependence, instead of the linear dependence occurring for $l_C \ll L \ll l_P, l_D$, when the tunnelling is non-coherent. The length dependence of the tunnelling resistance follows an L^{-2}-law.

In the investigation of the purely ballistic regime ($L \ll l_C$) we can neglect the collision integral in equation (10) and need not make the assumption about the smallness of the tunnelling matrix element which was essential for evaluation of the scattering-induced contributions in equations (14)–(17). The electron transport in coupled quantum wires in this regime is pertinent to the problem of electron-wave directional couplers. Theoretical studies of this problem [4–14], although rather extensive, included only a quantum mechanical calculation of the electronic transmission. Below, we show how the essential results of these studies can be obtained in a simple way from the quantum kinetic analysis. Integrating equation (10), with $\delta\hat{I}_{\pm}(\varepsilon, x) = 0$, over the energy and taking equation (13) into account, we find that the distribution of the chemical potentials is again described by equations (14)–(17) without the terms containing the scattering lengths l_C, l_P, and l_D. Since there is no backscattering, the solutions for $\mu^+_{l,r}$ and $\mu^-_{l,r}$ are decoupled:

$$\mu^+_{l,r}(x) = \mu_{1l,r} \mp (\mu_{1l} - \mu_{1r})r\sin^2(\Delta_T x/2\hbar v_F) \tag{42}$$

$$\mu^-_{l,r}(x) = \mu_{2l,r} \mp (\mu_{2l} - \mu_{2r})r\sin^2(\Delta_T(L-x)/2\hbar v_F). \tag{43}$$

Here $\Delta_T = (\Delta^2 + 4T^2)^{1/2}$ and $r = 4T^2/\Delta_T^2$. Equations (42) and (43) describe oscillations of the electronic wave packets between the layers due to coherent tunnelling. A complete

transfer of the wave packet can be achieved for $\Delta = 0$. One can calculate the resistance and transresistance as

$$R = \frac{\pi\hbar}{e^2}[1 - (r/2)\sin^2\psi]/[1 - r\sin^2\psi] \tag{44}$$

$$R_{TR} = -\frac{\pi\hbar}{2e^2} r\sin^2\psi/[1 - r\sin^2\psi] \tag{45}$$

and the tunnelling resistances as

$$R_{Ts} = R_{Tn} - \frac{\pi\hbar}{2e^2} = \frac{\pi\hbar}{2e^2} r^{-1}\sin^{-2}\psi. \tag{46}$$

Here $\psi = \Delta_T L/2\hbar v_F$. The oscillations of these quantities occur in a way similar to the one described by equations (38)–(40): $\delta_F L$ coincides with 2ψ if one replaces Δ by Δ_T. However, since the tunnel coupling is strong, the oscillations described by equations (44)–(46) have large amplitudes. In particular, when Δ is small ($r \simeq 1$), all the quantities given by equations (44)–(46) show giant oscillations with amplitude large in comparison to G_0^{-1}.

4. Conclusions

In this paper we carried out a theoretical study of electron transport in parallel 1D layers coupled by tunnelling and Coulomb interaction and contacted, at their ends, to quasi-equilibrium reservoirs. A linear-response, steady-state regime has been investigated, and the wires were assumed to be ideal, i.e., without defects and, therefore, the elastic scattering of electrons by them was neglected. As the most important result of our study, we found that a full quantum kinetic description of the problem is reduced, with physically reasonable assumptions, to a set of linear, first-order differential equations describing the distribution of local chemical potentials for forward- and backward-moving electrons. The boundary conditions for the chemical potentials are determined by the potentials of the reservoirs controlled by applied voltages. The solution of this set was obtained analytically and allowed us to describe the local currents flowing in each layer from the pure ballistic regime, when the electrons do not suffer any scattering along the wires, to the diffusive regime, when the electrons experience many backscattering events during the transport.

In particular, we applied our approach to the description of the resistance R, transresistance R_{TR}, and tunnelling resistances R_{Ts} and R_{Tn} of double quantum wires. The most important result is that R_{TR}, which is caused by both tunnelling and Coulomb-drag effects, depends on the wire length L non-monotonically and always changes its sign as L increases, because in shorter wires, when backscattering is rare, the tunnelling, as well as the drag, leads to a *negative* R_{TR}, while in longer wires, when the transport becomes diffusive, the tunnelling leads to a *positive* R_{TR}, in a way similar to that for coupled two-dimensional (2D) systems [28, 29], and overcomes the drag as the length L increases. This sign inversion is qualitatively understood from an analysis of the balance equation (28) and is mathematically described by equation (30). In the diffusive limit and for $l_T \gg l_P$, equations (29) and (30) formally coincide with those obtained previously [27, 29] for coupled 2D systems. Besides, some recent studies of transport phenomena in coupled 1D layers, namely transport without backscattering in tunnel-coupled edge states [30, 31] and Coulomb drag between quantum wires in the ballistic regime [21], constitute limiting cases of the more general results given by equations (29) and (30).

One should stress the importance of the phase-breaking processes that suppress the tunnel coherence. In our model, i.e., without elastic scattering, these processes are proved to be much more frequent than the backscattering processes. This allowed us to distinguish two transport regimes: the pure ballistic regime, without any scattering, and the pseudo-ballistic

one, without backscattering but with substantial forward scattering due to electron–electron interaction, and with suppressed coherence. If one considers just a single wire, there is no difference between these regimes as regards the electrical properties: the resistance is equal to G_0^{-1} in both cases. However, the electrical properties of a tunnel-coupled double-wire system behave differently as one passes from one regime to another, because the contribution of the tunnelling to the electrical properties becomes different. In the ballistic regime, as well as in the transition region between the two regimes ($L \sim l_C$), all calculated resistances oscillate with L and with the level-splitting energy Δ due to interference of the electron waves. One can vary the level-splitting energy by applying either a transverse voltage across the wires or a magnetic field perpendicular to the wire plane. In the latter case the oscillations show an Aharonov–Bohm periodicity associated with the magnetic flux penetrating through the area Lw between the wires. The oscillations become exponentially damped as the ratio L/l_C increases. If the tunnel coupling is strong, the oscillations have large amplitudes, which, from a theoretical point of view, can be much larger than the resistance quantum G_0^{-1}. So far the experimentally observed [9, 12] resistance oscillations in tunnel-coupled ballistic quantum wires have been of small ($\sim$0.5 kΩ) amplitude. This is not surprising because there are many factors which compete against the tunnel coherence. Apart from inelastic scattering considered in this paper, there are elastic scattering and long-scale inhomogeneities of the wires which would lead to a coordinate dependence of the level splitting Δ. If these variations of Δ were larger than the tunnelling matrix element, the coherence would be considerably suppressed.

We now discuss the approximations made in this paper. The main approximation is the neglect of elastic scattering. Since this scattering tends to be dominant at low temperatures, the presence of impurities in the 1D channels will considerably modify the transport. The elastic scattering will lead to an increase of backscattering and interference between the forward- and backward-moving electron waves. As a consequence, R, R_{TR}, R_{Ts}, and R_{Tn} will depend on the spatial positions of the impurities in the channels and the expressions obtained in this paper will not be valid. A further development of the transport theory for tunnel-coupled wires in the presence of elastic scattering is therefore desirable. On the other hand, advances in the technology of nanostructures, in particular selective doping, can make it possible to achieve structures where the elastic scattering in 1D channels is minimized for wire lengths smaller than a few microns which is the current standard of the impurity mean free path at low temperatures.

Another approximation concerns the transition from the quantum kinetic equation (1) to the semi-classical description given by equation (7). It is valid when the spatial scale of the electronic distribution is large in comparison to the Fermi wavelength $\pi\hbar/p_F$. We have seen that this scale is determined either by λ, given by equation (24) for long wires, or by $\lambda_\pm$, for short wires. In the case of strong tunnel coupling the characteristic scale is given by $\hbar v_F/\Delta_T$. Therefore, the necessary requirement is fulfilled if the tunnelling matrix element T, level splitting Δ, and the energy $\hbar/\tau_C$, associated with the smallest scattering time τ_C, are small in comparison to the Fermi energy. These conditions have been assumed throughout the paper. This also allowed us to neglect the difference between the electron densities in the layers and characterize the electrons in different layers by the same Fermi velocity $|v_p| \simeq v_F$.

The assumption about the adiabatic connection of the wires to the leads, which allowed us to neglect elastic scattering of electrons near the ends of the wires, implies that the Fermi wavelength $\pi\hbar/p_F$ must be small in comparison to the contact lengths, i.e., to the lengths of transition from the leads to the wires. On the other hand, the oscillations associated with the tunnel coherence, cf. section 3.2, can be seen if the contact lengths are smaller than both l_C and $\hbar v_F/\Delta_T$. In principle, both requirements can be fulfilled.

The next approximation, which allowed us to solve the kinetic equation analytically in the whole range of regimes from ballistic to diffusive, is equivalent to the following statement. In each layer the forward- and backward-moving electrons can be described as weakly coupled sub-systems characterized by their own local chemical potentials. This statement is obvious for the case of pure ballistic or pseudo-ballistic transport, when these potentials are merely dictated by the reservoirs (leads) and do not change with coordinate x. When the backscattering becomes important, this statement is still true if we assume that the forward-scattering events are much more frequent than the backscattering and tunnelling events. For example, it is always true for magnetic edge states, where one can completely neglect backscattering, and the introduction of local chemical potentials (see reference [31]) is well justified. In our case, a consideration of the electron–electron collision integral allowed us to estimate the characteristic time of the Coulomb-assisted forward scattering, and we find that it is of the order of τ_C, which is small in comparison with both backscattering times τ_P and τ_D. Thus, the electron–electron interaction provides an effective mechanism for forward scattering and can maintain quasi-equilibrium Fermi distribution functions for forward- and backward-moving electron sub-systems. However, these conditions may be violated when the conducting channels contain impurities with short-range potentials and the elastic backscattering becomes important.

Our evaluation of the characteristic scattering times from the collision integral has employed only the lowest-order essential contributions of the electron–phonon and electron–electron interactions, given by the diagrams of figure 2 and leading to collision integrals with scattering amplitudes in the Born approximation. While it is normally [32] good for electron–phonon interaction due to the weakness of the coupling constant, a rigorous evaluation of the electron–electron part requires also a consideration of higher-order contributions, given by more complex diagrams, because the ratio of the Bohr energy to the Fermi energy ε_F, which is the parameter of the perturbation expansion for the Coulomb interaction, is not small. Nevertheless, using the Born approximation in the evaluation of the drag time is still reasonable if the momentum $2p_F$ transferred in backscattering is large and the electron–electron backscattering probability is small. As regards τ_C, it is determined by forward-scattering processes with small momentum transfer and the Born approximation is not well justified [33]. On the other hand, our theory leads to a non-divergent expression (A.14) for τ_C and gives, for typical parameters of the electron system, physically reasonable values. We recall that in our theory both $k_B T_e$ and Δ are much smaller than ε_F. Therefore, one may expect equation (A.14) to provide a correct order-of-magnitude estimate of the time taken by the phase-breaking process caused by electron–electron interaction.

Finally, we stress that the results obtained in this paper hold for a normal Fermi-liquid state of the electron system. If the electrons in the wires are in the Luttinger-liquid state [34], these results have to be reconsidered.

Acknowledgment

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Appendix

Below, we give a microscopic calculation of the characteristic times τ_P, τ_D, and τ_C. The coordinate index x in the Green's functions and self-energies is omitted and $\hbar$ is set equal to 1. The normalization lengths are also set equal to 1. The electron–phonon self-energies given by

the diagrams shown in figure 2(a) are explicitly expressed as

$$\Sigma^{\alpha\beta}_{jj',\varepsilon}(p) = \mathrm{i}\sum_{Q}\int \frac{\mathrm{d}\omega}{2\pi}\, G^{\alpha\beta}_{jj',\varepsilon-\omega}(p-q)D^{\alpha\beta}_{jj'}(\omega, Q)M^{e-ph}_{jj}(\boldsymbol{Q})M^{e-ph}_{j'j'}(-\boldsymbol{Q}) \tag{A.1}$$

where the unperturbed Green's functions of phonons D^{-+} and D^{+-} (we do not need D^{--} and D^{++} in the following) are given as

$$\mathrm{i}D^{\mp\pm}_{jj'}(\omega, Q) = 2\pi\left[N_Q\delta(\omega \mp sQ) + (1+N_Q)\delta(\omega \pm sQ)\right] \tag{A.2}$$

and the matrix elements of the electron–phonon interaction are

$$\begin{aligned} M^{e-ph}_{jj}(\boldsymbol{Q}) &= \mathrm{i}\sqrt{E_1^2 Q/2\rho s}\, J^{e-ph}_j(q_y, q_z) \\ J^{e-ph}_j(q_y, q_z) &= \int\int \mathrm{d}y\ \mathrm{d}z\ F_j^2(y,z)\mathrm{e}^{\mathrm{i}q_y y + \mathrm{i}q_z z}. \end{aligned} \tag{A.3}$$

We use the expression sQ for the phonon energy, where $Q = |\boldsymbol{Q}|$, $\boldsymbol{Q} = (q, q_y, q_z)$ is the phonon wave vector, and s is the velocity of sound. Further, $N_Q = 1/[\exp(sQ/k_BT_e) - 1]$ is the Planck distribution function, ρ is the material density, and E_1 is the deformation potential constant.

The electron–electron self-energies given by the diagrams shown in figure 2(b) are expressed as

$$\begin{aligned} \Sigma^{\alpha\beta}_{jj'\varepsilon}(p) = 2(-1)^l \sum_{j_1 j_1'}\sum_{p',q} M^{e-e}_{jj_1}(q)M^{e-e}_{j'j_1'}(-q) \\ \times \int\int \frac{\mathrm{d}\omega\ \mathrm{d}\varepsilon'}{(2\pi)^2}\, G^{\alpha\beta}_{jj',\varepsilon-\omega}(p-q)G^{\beta\alpha}_{j_1'j_1,\varepsilon'}(p')G^{\alpha\beta}_{j_1j_1',\varepsilon'+\omega}(p'+q) \end{aligned} \tag{A.4}$$

where $l = 0$ for $\alpha = \beta$ and $l = 1$ for $\alpha \neq \beta$; the factor of 2 comes from the spin summation in the 'loop'. Here

$$M^{e-e}_{jj'}(q) = (2e^2/\epsilon)\int\int\int\int \mathrm{d}y\ \mathrm{d}y'\ \mathrm{d}z\ \mathrm{d}z'\ K_0(|q||\boldsymbol{r}-\boldsymbol{r}'|)F_j^2(y,z)F_{j'}^2(y',z') \tag{A.5}$$

are the matrix elements for electron–electron interaction, ϵ is the dielectric constant, K_0 is the modified Bessel function, and $|\boldsymbol{r}-\boldsymbol{r}'| = [(y-y')^2 + (z-z')^2]^{1/2}$.

For the evaluation of the collision integral we use equations (3)–(6) and express the non-equilibrium part of the matrix Green's functions according to (see also equation (9))

$$\delta\hat{G}^{\alpha\beta}_{\varepsilon}(\pm|p|) = \hat{g}^{\pm}_{\varepsilon}\hat{G}^{A}_{\varepsilon}(p) - \hat{G}^{R}_{\varepsilon}(p)\hat{g}^{\pm}_{\varepsilon}. \tag{A.6}$$

The collision integrals are evaluated below assuming weak tunnelling, when the non-diagonal contributions of $\hat{G}^{R,A}_{\varepsilon}(p)$ are neglected. This approximation is valid when the tunnelling matrix element T is small in comparison to the imaginary part of the self-energies and when the level splitting $|\Delta|$ is small in comparison to the Fermi energy. Both requirements are assumed fulfilled. Then the components $[\hat{g}^{\pm}_{\varepsilon}]_{jj'}$ enter only in the corresponding parts $\delta[\hat{I}(\varepsilon)]_{jj'}$ of the collision integral.

Now we calculate the *diagonal* parts for the electron–phonon scattering contribution to the collision integral. Taking the self-energy given by equations (A.1)–(A.3) we find

$$\int \mathrm{d}\varepsilon\ \delta[\hat{I}^{e-ph}_{+}(\varepsilon)]_{jj} = -(\mu^{+}_j - \mu^{-}_j)/2\tau_{Pj}$$

where $j = l, r$. The phonon-assisted transport time is given by

$$\begin{aligned} \tau^{-1}_{Pj} = \frac{E_1^2}{\rho s T_e}\sum_{q_y,q_z}|J^{e-ph}_j(q_y,q_z)|^2 \sum_{p,q\,(p>0,\,p-q<0)} v_p Q\int \mathrm{d}\varepsilon\ \frac{[f(\varepsilon) - f(\varepsilon - sQ)]}{4\sinh^2(sQ/2T_e)} \\ \times \left[G^{c}_{jj,\varepsilon-sQ}(p)G^{c}_{jj,\varepsilon}(p-q) + G^{c}_{jj,\varepsilon}(p)G^{c}_{jj,\varepsilon-sQ}(p-q)\right] \end{aligned} \tag{A.7}$$

where we defined $G^c = G^A - G^R$. For further evaluation of τ_{Pj} we use the free-particle (unperturbed) Green's functions, i.e.,

$$G^{R,A}_{ll,\varepsilon}(p) = [\varepsilon - \Delta/2 - p^2/2m \pm \mathrm{i}0]^{-1} \qquad G^{R,A}_{rr,\varepsilon}(p) = [\varepsilon + \Delta/2 - p^2/2m \pm \mathrm{i}0]^{-1}$$

and obtain

$$\tau_{Pj}^{-1} = \frac{E_1^2}{\rho s k_B T_e} \sum_{q_y,q_z} |J_j^{e-ph}(q_y,q_z)|^2 \times \int_{-\infty}^{\infty} \mathrm{d}q_1 \, \frac{Q[f(\mu + v_F q_1/2 - sQ/2) - f(\mu + v_F q_1/2 + sQ/2)]}{4\sinh^2(sQ/2k_BT_e)} \tag{A.8}$$

where $q_1 = q - 2p_F$ is a small variable. In the limit $k_BT_e \gg ms^2$ the integral over q_1 is easily calculated. In addition, if $k_BT_e \gg s[(\pi/a)^2 + (2p_F)^2]^{1/2}$, where a is the wire width, the scattering becomes quasi-elastic, and equation (A.8) is reduced to a known [26] result:

$$\tau_{Pj}^{-1} = \frac{2E_1^2 k_B T_e}{\rho s^2 v_F} \int\int \mathrm{d}y \, \mathrm{d}z \; F_j^4(y,z). \tag{A.9}$$

Since we assume that the confining potentials for the layers are almost identical, the difference between τ_{Pl} and τ_{Pr} is neglected: $\tau_{Pl} = \tau_{Pl} = \tau_P$. A numerical estimation, using equation (A.9) and GaAs material parameters, gives $\tau_P^{-1} \sim 10^{-2}\, k_BT_e$.

The electron–electron scattering contribution to the *diagonal* parts of the collision integrals gives the Coulomb-drag terms:

$$\int \mathrm{d}\varepsilon \; \delta[\hat{I}_{\pm}^{e-e}(\varepsilon)]_{jj} = -(\mu_j^{\pm} - \mu_j^{\mp})/2\tau_D + (\mu_{j'}^{\pm} - \mu_{j'}^{\mp})/2\tau_D$$

where $j' \neq j$. The drag time is given by

$$\tau_D^{-1} = \frac{4}{k_BT_e} \sum_{p,p',q} |M_{lr}^{e-e}(q)|^2 v_p \int\int\int \frac{\mathrm{d}\varepsilon \, \mathrm{d}\varepsilon' \, \mathrm{d}\omega}{(2\pi)^2} \, G^c_{ll,\varepsilon}(p) G^c_{ll,\varepsilon-\omega}(p-q) \times G^c_{rr,\varepsilon'}(p') G^c_{rr,\varepsilon'+\omega}(p'+q) \frac{[f(\varepsilon-\omega) - f(\varepsilon)][f(\varepsilon') - f(\varepsilon'+\omega)]}{4\sinh^2(\omega/2k_BT_e)}. \tag{A.10}$$

The sum here must be evaluated for $p > 0$, $p - q < 0$, $p' < 0$, and $p' + q > 0$. The evaluation of equation (A.10) using the free-particle Green's functions gives a simple result

$$\tau_D^{-1} = \frac{k_BT_e}{\pi v_F^2} |M_{lr}^{e-e}(2p_F)|^2 \frac{(\Delta/2k_BT_e)^2}{\sinh^2(\Delta/2k_BT_e)}. \tag{A.11}$$

One can estimate $M_{lr}^{e-e}(2p_F)$ as $(2e^2/\epsilon)K_0(2p_Fw)$, where w is the distance between the centres of the wires. If $2p_Fw \ll 1$, which is easily achieved for $w \sim 30$ nm, $K_0(2p_Fw)$ is exponentially small.

Finally, we calculate the electron–electron part of the *non-diagonal* components of the collision integral. Since the main contribution to it comes from the forward-scattering processes ($|q| \ll p_F$), only such processes are considered below. The integral of $[\hat{I}_{\pm}^{e-e}(\varepsilon)]_{jj'}$ ($j \neq j'$) over the energy ε can be reduced to a sum of three terms characterized by three different statistical factors:

$$\int \mathrm{d}\varepsilon \; \delta[\hat{I}_{\pm}^{e-e}(\varepsilon)]_{jj'} = -2 \int \mathrm{d}\varepsilon \; [g_{\varepsilon}^{\pm}]_{jj'} \sum_{p,p',q(p>0,p-q>0)} v_p \times \int\int \frac{\mathrm{d}\varepsilon' \, \mathrm{d}\omega}{(2\pi)^2} \big\{ (\Lambda_{j'j}^{ARAR} + \Lambda_{jj'}^{RARA})[f(\varepsilon')[1 - f(\varepsilon'+\omega)] + f(\varepsilon-\omega)[f(\varepsilon'+\omega) - f(\varepsilon')]] + (\Lambda_{j'j}^{ARRA} + \Lambda_{jj'}^{RAAR}) f(\varepsilon'+\omega)[1 - f(\varepsilon')] + (\Lambda_{j'j'}^{ARRA} + \Lambda_{jj}^{RAAR}) f(\varepsilon-\omega)[f(\varepsilon'+\omega) - f(\varepsilon')] \big\}. \tag{A.12}$$

In equation (A.12) we used the abbreviations

$$\Lambda_{jj'}^{\alpha\beta\gamma\delta} = G_{jj,\varepsilon}^{\alpha}(p)G_{j'j',\varepsilon-\omega}^{\beta}(p-q)\sum_{i} G_{ii,\varepsilon'}^{\gamma}(p')G_{ii,\varepsilon'+\omega}^{\delta}(p'+q)$$
$$\times \left[(M_{j'i}^{e-e}(q))^2 - M_{ji}^{e-e}(q)M_{j'i}^{e-e}(q)\right] \tag{A.13}$$

and neglected the terms with $\alpha = \beta$ and $\gamma = \delta$ because they vanish upon carrying out the summations over p and p', respectively. Since $[g_{\varepsilon}^{\pm}]_{lr} = [g_{\varepsilon}^{\pm}]_{rl}^{*} = g_{u,\varepsilon}^{\pm} - \mathrm{i}g_{v,\varepsilon}^{\pm}$, one can see that

$$\int \mathrm{d}\varepsilon\ \delta[\hat{I}_{\pm}^{e-e}(\varepsilon)]_{jj'} = \int \mathrm{d}\varepsilon\ \delta[\hat{I}_{\pm}^{e-e}(\varepsilon)]_{j'j}^{*}.$$

Calculating the integrals in the expression (A.12) within the approximation of the free-particle Green's functions, we find that the third term on the right-hand side of equation (A.12) vanishes. The first term diverges for $\Delta = 0$ but it is close to zero for $\Delta \neq 0$ and can be neglected in the following. In contrast, the second term gives a contribution, which can be represented, on account of equation (13), as

$$\int \mathrm{d}\varepsilon\ \delta[\hat{I}_{\pm}^{e-e}(\varepsilon)]_{jj'} = -\mu_{jj'}^{\pm}/\tau_C.$$

The 'non-diagonal' relaxation time τ_C (we take into account only its *real* part) is given by

$$\tau_C^{-1} = \frac{e^4 S^2 \Delta}{2\pi\epsilon^2 v_F^2} \coth\frac{\Delta}{4k_B T_e} \tag{A.14}$$

where

$$S = -\int\int\int\int \mathrm{d}y\ \mathrm{d}y'\ \mathrm{d}z\ \mathrm{d}z'\ \ln|\boldsymbol{r} - \boldsymbol{r}'|F_l^2(y,z)[F_l^2(y',z') - F_r^2(y',z')]. \tag{A.15}$$

In the calculation we took into account that $M_{ll}^{e-e}(q) \simeq M_{rr}^{e-e}(q)$ and $qa \ll 1$. The last property allowed us to use the approximation $K_0(x) \simeq -[C + \ln(x/2)]$, where C is Euler's constant; we found $M_{ll}^{e-e}(q) - M_{lr}^{e-e}(q) \simeq (2e^2/\epsilon)S$. The overlap integral S can be approximated, to a good accuracy, by $\ln(w/a)$.

If $\Delta \gg 4k_B T_e$, the relaxation rate τ_C^{-1} given by equation (A.14) is temperature independent and proportional to $|\Delta|$. For $\Delta \ll 4k_B T_e$, τ_C^{-1} is proportional to T_e. A comparison of equation (A.11) and equation (A.14) shows that τ_C is always much smaller than τ_D, since τ_C is controlled by forward-scattering processes and does not exhibit the smallness associated with the factor $[K_0(2p_F w)]^2$. A numerical estimation also shows that $\tau_C \ll \tau_P$, because of the weakness of the electron coupling to acoustical phonons. For this reason we neglected the contribution of electron–phonon scattering to the non-diagonal part of the collision integral.

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